

Trace anomalies and the $\Delta I = \frac{1}{2}$ rule

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Abstract

Trace Anomaly Dominance in weak K -decays successfully reproduces the $\Delta I = \frac{1}{2}$ selection rule results, as observed in $K_S \rightarrow \pi\pi$, $K_L \rightarrow \pi\pi\pi$, $K_S \rightarrow \gamma\gamma$ and $K_L \rightarrow \pi^0\gamma\gamma$.

1 Introduction

A precise quantitative understanding of the $\Delta I = \frac{1}{2}$ selection rule in K decays still remains an elusive goal. Indeed the short-distance evolution of $\Delta S = 1$ dimension six weak operators [1][2] cannot by itself reproduce the huge enhancement of the isospin $I = 0$ component of the $K^0 \rightarrow \pi\pi$ decay amplitudes relative to the $I = 2$ one.

Moreover, pursuing the operator evolution through quantum loops regularized within the truncated non-linear σ -model [3], the chiral quark model [4] or the extended Nambu and Jona-Lasinio model [5] introduces large uncontrollable theoretical uncertainties mainly due to the matching procedure between the (short-distance) Wilson coefficients and the (long-distance) hadronic matrix elements.

Finally, these phenomenological approaches do not shed light on the possible contribution of effective hadronic operators which are not directly accessible through perturbative QCD corrections to the Fermi current-current Hamiltonian.

As a consequence, the dominant effective Hamiltonian for $\Delta S = 1$ hadronic weak decays remains somewhat heuristic.

In this Letter we advocate a specific non perturbative effect, namely the *trace anomaly*, as the mechanism responsible for the bulk of the empirical $\Delta I = \frac{1}{2}$ rule.

We will show that the QCD trace anomaly, already known to dominate in the $\Psi' \rightarrow J/\Psi\pi\pi$ decay [6], also gives rise to a large non perturbative $\Delta S = 1$ weak operator which adequately describes $K \rightarrow 2\pi, 3\pi$ decays. The QED trace anomaly then allows for a parameter free calculation of the decays $K_S \rightarrow 2\gamma$ and $K_L \rightarrow \pi\gamma\gamma$ which agrees with the data.

Before concluding this Letter we briefly comment on the relation and the differences of our somewhat unconventional approach with more traditional points of view.

2 Trace anomalies

The energy momentum tensor $T_{\mu\nu}$ of a quantum field theory is most easily obtained [7] from the variation of its action

$$S = \int d^4x \sqrt{-g} \mathcal{L}(x) \quad (1)$$

with respect to a non-trivial space-time metric $g^{\mu\nu}$

$$\delta S \equiv \frac{1}{2} \int d^4x \sqrt{-g} T_{\mu\nu}(x) \delta g^{\mu\nu}(x). \quad (2)$$

If we represent the scale transformation

$$x^\mu \rightarrow e^\lambda x^\mu \quad (3)$$

as a particular change in the metric

$$g^{\mu\nu}(x) \rightarrow e^{-2\lambda} g^{\mu\nu}(x) \quad (4)$$

then the corresponding change in the Lagrangian $\mathcal{L}(x)$ is proportional to the trace T of $T_{\mu\nu}$. In other words, scale invariance requires a traceless energy-momentum tensor.

It is also well-known that the classical scale invariance of gauge theories with massless matter fields is broken by quantum corrections. Indeed, an infinitesimal scale transformation induces a shift in the renormalized gauge coupling

$$g(\mu) \rightarrow g(e^{-\lambda}\mu) = g(\mu) - \lambda\beta(g) \quad (5)$$

such that the corresponding change in the Lagrangian is

$$\mathcal{L}(g) \rightarrow \mathcal{L}(g) - \lambda \beta(g) \frac{\partial \mathcal{L}}{\partial g}. \quad (6)$$

Consequently, a trace anomaly arises for the classically conserved dilatation current:

$$\begin{aligned} \partial_\mu (T^{\mu\nu} x_\nu) &\equiv T^{(m=0)} \\ &= \beta(g) \frac{\partial \mathcal{L}}{\partial g}. \end{aligned} \quad (7)$$

For chiral QCD, the trace anomaly reads

$$T_{\text{QCD}}^{(m=0)} = \frac{\beta(g_s)}{2g_s} G_{\mu\nu}^a G_a^{\mu\nu} \quad (a = 1, \dots, 8) \quad (8)$$

and a similar expression holds for $T_{\text{QED}}^{(m=0)}$ as well.

Although trace anomalies are intrinsically non perturbative, it is nevertheless useful for our purposes to give their explicit expression to lowest order in α_s and in α respectively. For three flavours of massless quarks this leads to

$$\begin{aligned} T^{(m=0)} &= T_{\text{QCD}}^{(m=0)} + T_{\text{QED}}^{(m=0)} \\ &\cong -\frac{9}{8} \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_a^{\mu\nu} + \frac{1}{3} \frac{\alpha}{\pi} F_{\mu\nu} F^{\mu\nu}. \end{aligned} \quad (9)$$

On the other hand, if we consider the low energy effective lagrangian for the octet π of pseudoscalar Goldstone bosons

$$\mathcal{L}_{\text{eff}} = \frac{f^2}{8} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) \quad (10)$$

with

$$U = \exp i\sqrt{2} \frac{\pi}{f} \quad , \quad f = 132 \text{ MeV} \quad (11)$$

the effective strong anomaly is easily computed and the total trace anomaly is now given by

$$\begin{aligned} T_{\text{eff}}^{(m=0)} &= T_{\text{eff, strong}}^{(m=0)} + T_{\text{QED}}^{(m=0)} \\ &= -\frac{f^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + T_{\text{QED}}^{(m=0)}. \end{aligned} \quad (12)$$

3 The QCD trace anomaly and hadronic K^0 decays

From Eqs(9) and (12), the gluon conversion into a (low-energy) system of two pions in a relative s -wave is calculable from first principles and

$$\langle \pi^+ \pi^- | T_{\text{QCD}}^{(m=0)} | 0 \rangle = (p_+ + p_-)^2. \quad (13)$$

This remarkable property of the strong interactions was elegantly exploited [6] to predict the two pion invariant mass distribution in $\Psi' \rightarrow (J/\Psi) \pi^+ \pi^-$ decays. The new experimental data from the BES collaboration [8] beautifully confirm this prediction.

The physical picture underlying this exclusive Ψ' decay is thus a simple two-step process: emission of soft gluons (in a 0^{++} state) from a *heavy* quark and then, via the trace anomaly, hadronization of these gluons into a pair of pions.

It is of course tempting to invoke a similar mechanism in hadronic K decays, namely emission of soft gluons (in a 0^{++} state) by a *light* quark and again hadronization via the trace anomaly. In such a case, the dominant $\Delta S = 1$ effective Hamiltonian at low energy would thus read

$$\mathcal{H}_{\text{TAD}}^{\Delta S=1} = g_8 r (M U^\dagger + U M^\dagger)^{ds} T_{\text{eff, strong}}^{(m=0)} \quad (14)$$

where $M = \text{diag}(m, m, m_s)$ is the light quark mass matrix responsible for the pseudoscalar squared masses $m_\pi^2 = rm$ and $m_K^2 = \frac{r}{2}(m + m_s)$, in the isospin limit.

The weak Hamiltonian Eq.(14) has the correct behaviour under both chiral $SU(3)_L \otimes SU(3)_R$ and CPS [9] transformations. The resulting $\Delta I = \frac{1}{2}$ hadronic decay amplitudes are given by¹

$$\begin{aligned} A(K_S \rightarrow \pi^+ \pi^-) &= i \frac{4\sqrt{2}}{f} g_8 (m_K^2 - m_\pi^2) m_K^2 \\ A(K_L \rightarrow \pi^+ \pi^0 \pi^-) &= \frac{4}{f^2} g_8 m_K^2 \left(\frac{1}{3} m_K^2 + m_\pi^2 Y \right) \end{aligned} \quad (15)$$

with

$$Y = \frac{(s_3 - s_0)}{m_\pi^2}, \quad s_0 = \frac{1}{3}(s_+ + s_3 + s_-)$$

the standard Dalitz variables. These amplitudes turn out to be identical to the ones obtained from the conventional chiral Hamiltonian

$$\mathcal{H}_\chi^{\Delta S=1} = \frac{f^4}{4} G_8 (\partial_\mu U \partial^\mu U^\dagger)^{ds} \quad (16)$$

provided we make the following identification

$$G_8 = 4 \frac{m_K^2}{f^2} g_8. \quad (17)$$

In particular, we obtain a reasonable (linear) fit of the $K_L \rightarrow \pi\pi\pi$ Dalitz plot in terms of the g_8 parameter extracted from the measured $K \rightarrow \pi\pi$ decay widths:

$$|g_8^{\text{exp}}| = 0.16 \cdot 10^{-6} \text{Gev}^{-2}. \quad (18)$$

4 The QED trace anomaly and radiative K^0 decays

If our hypothesis of Trace Anomaly Dominance in the $\Delta I = \frac{1}{2}$ K^0 decays holds true, it is straightforward to extend Eq.(14) to radiative processes by simply replacing $T_{\text{eff, strong}}^{(m=0)}$

¹Here and in what follows, we always assume CP invariance, i.e. real g_8 .

by $T_{\text{QED}}^{(m=0)}$.

The resulting K_S decay amplitude into two real photons is then

$$A(K_S \rightarrow \gamma\gamma) = i \frac{16\sqrt{2}}{3f} g_8 (m_K^2 - m_\pi^2) \frac{\alpha}{\pi} \left[(q_1 \cdot q_2)(\varepsilon_1 \cdot \varepsilon_2) - (q_1 \cdot \varepsilon_2)(q_2 \cdot \varepsilon_1) \right]. \quad (19)$$

From this equation and Eqs(15), it follows that

$$\begin{aligned} \text{Br}(K_S \rightarrow \gamma\gamma) &= \left(\frac{2\alpha}{3\pi} \right)^2 \left(1 - 4 \frac{m_\pi^2}{m_K^2} \right)^{-\frac{1}{2}} \text{Br}(K_S \rightarrow \pi^+ \pi^-) \\ &= (2.0 \pm 0.2) 10^{-6}. \end{aligned} \quad (20)$$

The quoted error in Eq.(20) corresponds to our neglect of (small) $\Delta I = \frac{3}{2}$ contributions.

This result is in good agreement with the recent measurement of NA48 Collaboration [10]

$$\text{Br}(K_S \rightarrow \gamma\gamma) = (2.6 \pm 0.5) 10^{-6}. \quad (21)$$

In a similar way, we consider the $K_L \rightarrow \pi^0 \gamma\gamma$ radiative decay mode. The corresponding amplitude is simply given by

$$A(K_L \rightarrow \pi^0 \gamma\gamma) = \frac{16}{3f^2} g_8 m_K^2 \frac{\alpha}{\pi} \left[(q_1 \cdot q_2)(\varepsilon_1 \cdot \varepsilon_2) - (q_1 \cdot \varepsilon_2)(q_2 \cdot \varepsilon_1) \right]. \quad (22)$$

From Eqs(19), (21) and (22), we obtain the trace anomaly induced branching

$$\begin{aligned} \text{Br}(K_L \rightarrow \pi^0 \gamma\gamma) &\approx 0.49 \text{ Br}(K_S \rightarrow \gamma\gamma) \\ &= (1.3 \pm 0.3) 10^{-6} \end{aligned} \quad (23)$$

in fair agreement with the world average value [11]

$$\text{Br}(K_L \rightarrow \pi^0 \gamma\gamma) = (1.68 \pm 0.10) 10^{-6}. \quad (24)$$

In our approach based on the dominance of trace anomalies, the 2γ invariant mass distribution in $K_L \rightarrow \pi^0 \gamma \gamma$ is predicted to be negligible at low z : with

$$z \equiv \frac{(q_1 + q_2)^2}{m_K^2} \quad (25)$$

the spectrum is indeed given by

$$\frac{d\Gamma}{dz} = \frac{1}{36\pi^3 f^4} g_8^2 m_K^9 z^2 \lambda^{\frac{1}{2}} \left(1, z, \frac{m_\pi^2}{m_K^2} \right) \quad (26)$$

where $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2(xy + xz + yz)$ is the usual kinematical function².

5 Trace Anomaly Dominance versus Chiral Hamiltonians

In Section 3, we already pointed out that the conventional chiral Hamiltonian $\mathcal{H}_\chi^{\Delta S=1}$ and the trace anomaly Hamiltonian $\mathcal{H}_{\text{TAD}}^{\Delta S=1}$ give identical results for hadronic K decays.

The two radiative K decays, on the other hand, are often presented as significant tests of chiral perturbation theory: pion loops generated by the chiral Hamiltonian allow these processes to occur. But for $\mathcal{H}_{\text{TAD}}^{\Delta S=1}$, two photons are *directly* a piece of the trace anomaly.

For the decay $K_S \rightarrow 2\gamma$, the physics of the two pictures is not that different: the pion loop is indeed an “effective scalar” just as the trace anomaly.

For $K_L \rightarrow \pi^0 \gamma \gamma$, chiral perturbation theory meets with some difficulty to account for the measured branching ratio [13] but not to reproduce the 2γ energy spectrum distribution. Once again the latter fact follows from an effective scalar coupling of the two photons.

²We use the same notation as in [12].

Whether the rate problem of $K_L \rightarrow \pi^0 \gamma \gamma$ is a serious shortcoming of $\mathcal{H}_\chi^{\Delta S=1}$ remains to be seen but, in any case, $\mathcal{H}_{\text{TAD}}^{\Delta S=1}$ does agree with all experimental information presently available.

What, then, is the dominant effective Hamiltonian for $\Delta S = 1$ weak decays? $\mathcal{H}_\chi^{\Delta S=1}$, $\mathcal{H}_{\text{TAD}}^{\Delta S=1}$ or a linear combination of both?

The short-distance evolution [1] of the single $\Delta I = \frac{1}{2}$ four-quark operator

$$Q_2 - Q_1 \equiv L_\mu^{su} L_{ud}^\mu - L_\mu^{sd} L_{uu}^\mu \quad (27)$$

with

$$L_\mu^{ij} \equiv \bar{q}^i \gamma_\mu (1 - \gamma_5) q^j$$

down to the charm mass scale obviously favours $\mathcal{H}_\chi^{\Delta S=1}$. Indeed, the hadronized left-handed current derived from the low-energy effective Lagrangian given in Eq.(10) reads

$$L_\mu^{ij} = \frac{if^2}{2} (\partial_\mu U U^\dagger)^{ji}. \quad (28)$$

However, such a hadronization requires first to evolve further down. But below the charm mass scale, the GIM cancellation mechanism is not efficient anymore and penguin-like diagrams [2] involving charge $\frac{2}{3}$ quark loops arise. Among them, an “annihilation-penguin” diagram with the *heavy* charm quark running inside the loop induces an effective $\mathcal{H}_{\text{TAD}}^{\Delta S=1}$. The resulting perturbative estimate [14] gives a negligible contribution to the $\Delta I = \frac{1}{2}$ rule

$$\begin{aligned} g_8^{\text{SD}} &= \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \frac{f^2}{1080 m_c^2} \\ &\approx 10^{-4} g_8^{\text{exp}}. \end{aligned} \quad (29)$$

But the *same* topology with now a “soft” up quark running in the loop will induce a sizeable enhancement of the $\Delta I = \frac{1}{2}$ component of the $K_S \rightarrow \pi\pi$ decay amplitude. This is supported by an estimate based on QCD sum rules [15].

It seems fair to conclude that we do not have any a priori theoretical reason for choosing between $\mathcal{H}_\chi^{\Delta S=1}$ and/or $\mathcal{H}_{\text{TAD}}^{\Delta S=1}$. Of course, a reliable non perturbative estimate of G_8 and g_8 would settle the question !

6 Conclusion

We have argued that trace anomalies might in fact dominate the $\Delta I = \frac{1}{2}$ component of K^0 decay amplitudes. This rather unconventional approach based on the effective Hamiltonian defined in Eq.(14) directly predicts $K_S \rightarrow \pi\pi$, $K_L \rightarrow \pi\pi\pi$, $K_S \rightarrow \gamma\gamma$ and $K_L \rightarrow \pi^0\gamma\gamma$ decay amplitudes in terms of a single parameter g_8 . All predictions are in good agreement with the data.

The Trace Anomaly Dominance works beautifully in $\Psi' \rightarrow (J/\Psi)\pi\pi$ decays, and has been suggested [16] as a possible explanation of the so-called “ $\rho\pi$ puzzle”. For the $\Delta I = \frac{1}{2}$ rule in K -decays, its contribution cannot be ignored anymore.

A deeper understanding of the $\Delta I = \frac{1}{2}$ selection rule appears to be at hand: it depends on (non perturbative) estimates of our parameter g_8 as well as the parameter G_8 of the conventional chiral Hamiltonian. In principle, such an estimate is accessible to lattice gauge theory.

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